Inclined Plane

Equipment

- Calculator, Computer
- A book with a hard cover (physics textbook?)
- A quarter
- Stopwatch

Objectives

- *To determine a coefficient of static friction between a metal and paper surfaces*
- *To determine the acceleration of an object moving along an incline*
- *To use the acceleration of an object to calculate the coefficient of kinetic friction*

Introduction

In an earlier experiment, you attempted to use a stopwatch to time the fall of a dropped object. It was almost impossible to get a good result by this method because the time of drop was almost as much as the reaction time of a person working the stopwatch. In this lab, we will be using the stopwatch again, but to slow down the action, the object (a quarter) will glide down an incline instead of falling straight down.

You already know the basics of kinematics. When an object undergoes one-dimensional uniformly accelerated motion, its velocity increases linearly with time. If ***the initial velocity of the object is zero at time*** $\boldsymbol{t = 0}$, then the velocity v at any time t is given by the following expression:

$$v(t) = at \tag{1}$$

Here, a is the acceleration, which is constant in magnitude and direction.

The displacement x of the object during the time interval t is given by:

$$x(t) = \frac{1}{2}at^2 \tag{2}$$

Thus, Equation 2 states that if an object is released from rest, its displacement is directly proportional to the square of the elapsed time.

When an object is dropped, we assume **free fall**, which means its acceleration is g in a downward direction. A closely related situation is an object gliding down a slope. In this case, the acceleration is reduced by a factor determined by the angle of the slope. This results from the fact that the force of gravity $\vec{F}_g$ is a vector which can be resolved into two components, one component is perpendicular to the inclined surface and another component is parallel to the inclined surface. The perpendicular component is offset by the support force from the surface. The parallel component $(F_g \sin\theta)$ pulls the object down the inclined surface. If the value of the parallel component $(F_g \sin\theta)$ is below the limit of the static friction force, there will be no motion despite the pull because the static friction force will match the value up to the limit.

Therefore, for an object placed on the inclined plane but not moving because the component of the gravitational force parallel to the surface equals the limit of the static friction force:

$$F_g \sin\theta = \mu_s n \tag{3}$$

Where n is the normal force exerted on the object the surface in the direction perpendicular to the surface. Because the object in equilibrium, the normal force and the component of gravitational force perpendicular to the inclined surface are balancing each other out. Hence,

$$\begin{gathered} F_g \cos\theta = n, \text{ replacing in equation (3)} \\ F_g \sin\theta = \mu_s F_g \cos\theta \end{gathered} \tag{4}$$

After canceling out the gravitational force and dividing both sides by $\cos\theta$,

$$\mu_s = \frac{sin\theta}{cos\theta} = tan\theta_{limit} \tag{5}$$

However, if the angle of the incline exceeds the limit, the component of the gravitational force parallel to the surface will bridge the limit of the static friction force and the object will slide along the inclined surface with an acceleration. Per 2nd Law of Motion:

$$\begin{gathered} ma = mg\sin\theta - \mu_k mg\cos\theta \\ a = g\sin\theta - \mu_k g\cos\theta, \text{ after re-arranging} \\ \mu_k = \frac{g\sin\theta - a}{gcos\theta} \end{gathered} \tag{6}$$

By timing the slide of the coin over a set distance, the acceleration of the coin can be calculated using kinematics equation (2). Then, the calculated value of the acceleration along with the measured value of the angle can be used in computation of the coefficient of kinetic friction while utilizing equation (6).

Experimental Setup

A coin (quarter) placed on the hard book cover (Figure 1) will not slide when the cover is lifted to open the book until the angle reaches the limit of static friction.

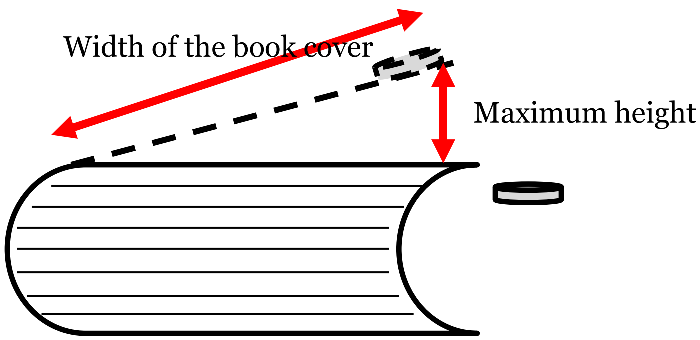

Figure 1. A quarter coin on the edge of the book cover

Part 1: Determining the coefficient of static friction

1. Place a coin (quarter) on the edge of a hardcover book.
2. Measure the width of the book cover using any units but convert the result to meters before recording it in Table 1.
3. Gradually lift the cover to open the book until the coin starts sliding.
4. Measure the maximum height above the pages when the coin is not sliding.
5. Calculate the maximum angle from the measure height and width of the book cover by computing $\theta_{limit} = \sin^{-1}(\frac{\text{maximum height}}{\text{width of the book cover}})$.
6. Calculate the coefficient of static friction by equation (5)

	Partner 1	**Partner 2**	**Partner 3**
Condition of surface	***Smooth***	***Coarse***	***Glossy***
Width of the book cover (m)			
Maximum height above the pages (m)			
$\boldsymbol{\theta_{limit}}$			
$\boldsymbol{\mu_s}$			

Table 1. Measurements and calculations to determine the coefficient of static friction

Part 2: Calculation the coefficient of kinetic friction

1. Lift the cover to open the book above the $\boldsymbol{\theta_{limit}}$
2. Secure the position of the cover and measure the height above the pages.
3. Calculate the angle (follow step 5 of Part 1)
4. Release the coin from the edge of the cover and time the slide.
5. Keeping the same angle, repeat the slide 4 more times and average the results.
6. Calculate the acceleration of the coin by re-arranging equation (2)
7. Calculate the coefficient of kinetic friction by using equation (6)
8. Make a 2-series bar diagram presenting both, the coefficient of static and kinetic friction, for each surface.

		Partner 1	**Partner 2**	**Partner 3**
Condition of surface		***Smooth***	***Coarse***	***Glossy***
Width of the book cover **(meters!)**				
Height above the pages (**meters!)**				
$\boldsymbol{\theta_{sliding}}$				
Sliding time (s)	Trial 1			
	Trial 2			
	Trial 3			
	Trial 4			
	Trial 5			
Average Time of the slide (s)				
Acceleration (m/s²)				
$\boldsymbol{\mu_k}$				

Table 2. Measurements and calculations to determine the acceleration of the

Requirements for the Report (also consult the rubric):

Only one report needs to be submitted per lab group from the Blackboard "Report Submissions" page. The names of the lab partners collaborating on the report must be listed in the header on each page of the report; a lab partner whose name is not listed will not receive the credit unless separate report is submitted individually (such report will have 10 points deduction for the team work). If there was a "division of labor" between lab partners, the report must include a credits section at the end listing the authors of each section. (Example: The Excel graphs were done by ….; the Data tables were done by …; the abstract was written by …)

- The **abstract section** must contain the following explanations in paragraph form:
  - How the data was collected and calculated for Table 1 including formulas used in calculations
  - How the data was collected and calculated for Table 2 including formulas used in calculations
  - How the data from both, Table 1 and Table 2 was analyzed with a bar diagram.
  - A general statement based off the bar diagram comparing the values of kinetic and static coefficients for *the same type of the surface* AND reflecting on how the condition of the surface is represented in the value of the friction coefficients.

- The **data section** must include
  - **2 Tables** (labeled and captioned)
  - **1 Bar diagram** (titled, with legend, and bars labeled; figure should be labeled and captioned)
    - Series 1: coefficient of static friction vs. condition of the surface
    - Series 2: coefficient of kinetic friction vs. condition of the surface.